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LETTER TO THE EDITOR

Complete solution of the kinetics in a far-from-equilibrium Ising chain

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Abstract
The one-dimensional Ising model is easily generalized to a *genuinely nonequilibrium* system by coupling alternating spins to two thermal baths at different temperatures. Here, we investigate the full time dependence of this system. In particular, we obtain the evolution of the magnetization, starting with arbitrary initial conditions. For slightly less general initial conditions, we compute the time dependence of all correlation functions, and so, the probability distribution. Novel properties, such as oscillatory decays into the steady state, are presented. Finally, we comment on the relationship with a reaction–diffusion model with pair annihilation and creation.

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Introduction

With their connections to both fundamental issues of statistical mechanics and applications to a range of disciplines, nonequilibrium many-body systems have received much attention recently (see e.g. [1] and references therein). Despite these efforts a comprehensive theoretical approach is still lacking: As yet, there is no equivalent of the Gibbs ensemble theory for nonequilibrium systems. Consequently, much of the observed macroscopic properties of such systems are sensitive to the underlying microscopic dynamics, in contrast to systems in thermal equilibrium. In particular, most progress in this field is made by studying paradigmatic models [1], with a master equation governing their evolution. In this context, exact solutions of simple models are very valuable (but also very rare), as they can be used as milestones to develop approximate/numerical schemes and to shed light on some general properties of related models. The Ising model is a good example with a venerable history [1, 2]. Recently, an interesting generalization of it was studied [3]: a kinetic Ising chain in which spins at alternating sites are coupled to thermal baths of two *different* temperatures. As a result, at long times, this system reaches a stationary state with inherently *nonequilibrium* properties, e.g., a nonzero heat flux through the system [3]. Subsequently, *all* correlation functions were computed exactly, so that the full stationary distribution is known [4, 5]. Various other versions of two- or multiple-temperature models have been studied [6].

In this letter we show that the *dynamic* aspects of this model are also accessible. As illustrations, we present the complete solution of the time-dependent magnetization and two-spin correlation function. These quantities can then be exploited to compute *all* other correlation functions, so that the time-dependent probability distribution is also (at least formally) known. The key behind the solvability of this system lies in the simple structure of its Glauber-like kinetics, so that the usual BBGKY hierarchy [7] decomposes into a closed set of *linear* equations for each N-spin correlation function.

Model specifications

We consider an Ising spin chain defined on a ring of L sites. For simplicity, we choose L to be even and denote a spin at site j by σ_j (which assumes values ± 1). The spins interact ferromagnetically via the usual nearest-neighbour Hamiltonian: $\mathcal{H} = -J\sum_j \sigma_j\sigma_{j+1}$ ($J > 0$; the anti-ferromagnetic case can be accessed by a gauge transformation). Next, we endow the system with a Glauber-like dynamics, but couple spins on the even and odd sites to reservoirs at temperatures T_e and T_o, respectively. For $T_e \neq T_o$, this dynamics violates detailed balance [3–5] and leads to a nonequilibrium stationary (but probably non-Gibbsian) state. Denoting a configuration $(\sigma_1, \sigma_2, \ldots, \sigma_L)$ of our system by $\{\sigma\}$, we implement this dynamics through a master equation for time-dependent probability distribution $P(\{\sigma\}, t)$:

$$\partial_t P(\{\tilde{\sigma}\}, t) = \sum_{\{\sigma\}} [W(\{\tilde{\sigma}\}; \{\sigma\})P(\{\sigma\}, t) - W(\{\sigma\}; \{\tilde{\sigma}\})P(\{\tilde{\sigma}\}, t)] \tag{1}$$

with transition rates

$$W(\{\tilde{\sigma}\}; \{\sigma\}) = \sum_j \frac{1}{2}\left[1 + \gamma_j\tilde{\sigma}_j\left(\frac{\sigma_{j-1} + \sigma_{j+1}}{2}\right)\right]\prod_{k\neq j}\delta(\tilde{\sigma}_k, \sigma_k). \tag{2}$$

Here, γ_j is $\gamma_e \equiv \tanh(2J/k_bT_e)$ for even j and $\gamma_o \equiv \tanh(2J/k_bT_o)$ for odd j and δ is the Kronecker delta. For convenience, the overall factor of $1/2$ is chosen so that all decays follow a simple e^{-t}-law in the $J = 0$ limit.

Our goal is to compute *all* correlation functions $\left\langle\sigma_{j_1}\cdots\sigma_{j_n}\right\rangle_t \equiv \sum_{\{\sigma\}}\sigma_{j_1}\ldots\sigma_{j_n}P(\{\sigma\}, t)$ and to represent the complete solution for $P(\{\sigma\}, t)$ by the relation [2]

$$2^L P(\{\sigma\}, t) = 1 + \sum_i \sigma_i\langle\sigma_i\rangle_t + \sum_{j<k}\sigma_j\sigma_k\langle\sigma_j\sigma_k\rangle_t + \sum_{j<k<l}\sigma_j\sigma_k\sigma_l\langle\sigma_j\sigma_k\sigma_l\rangle_t + \cdots. \tag{3}$$

Recently, the stationary distribution, $P(\{\sigma\}, t = \infty)$, was found in this manner [4]. We will first present the solutions for $\langle\sigma_i\rangle_t$ and $\langle\sigma_j\sigma_k\rangle_t$ and then, in terms of these, provide expressions for the other correlations.

For Ising chains in contact with only one thermal bath, it is well known that a gauge transformation (changing the sign of every other spin) relates a system coupled to $T < 0$ to one coupled to $T > 0$. Here, it is clear that such a transformation is applicable if the signs of *both* T_e and T_o are changed. Thus, we will investigate explicitly two cases: one when both T's are positive and the other, when they are of opposite signs. As noted in [3], quite unusual properties arise in the latter case. Here, we will find similar oscillatory behaviour, but in the time domain.

The time-dependent magnetization

The single-spin function $\langle\sigma_j\rangle_t$ is, of course, just the t-dependent magnetization at site j, which we denote by $m_j(t)$. With the master equation (1), the equation of motion of the local magnetization reads

$$\frac{\mathrm{d}}{\mathrm{d}t}m_j(t) = \frac{\gamma_j}{2}[m_{j-1}(t) + m_{j+1}(t)] - m_j(t). \tag{4}$$

Given any initial $m_j(0)$, the full t-dependent magnetization is just

$$m_j(t) = \sum_k M_{jk}(t)m_k(0),$$

where $M_{jk}(t)$ is the 'propagator'. In equilibrium ($\gamma_j = \gamma$), Glauber [2] obtained $M_{jk}(t) = \mathrm{e}^{-t}I_{k-j}(\gamma t)$, where $I_n(t)$ denotes usual modified Bessel function [8]. Though our system is not in equilibrium, we exploit this result by defining a modified magnetization, $m_j(t)/\sqrt{\gamma_j}$, which allows us to reduce (4) to the Glauber case with γ replaced by $\alpha \equiv \sqrt{\gamma_e\gamma_o}$, the same parameter that enters into the steady-state correlations functions in [3, 4]. In other words, provided $T_e, T_o > 0$, we can associate our system with an equilibrium one, coupled to a bath with temperature T_{eff}, given by $\tanh[2J/k_bT_{\mathrm{eff}}] = \sqrt{\tanh[2J/k_bT_e]\tanh[2J/k_bT_o]}$. To be precise, we have

$$M_{jk}(t) = \mathrm{e}^{-t}\sqrt{\frac{\gamma_j}{\gamma_k}}I_{k-j}(\alpha t); \qquad \alpha \equiv \sqrt{\gamma_e\gamma_o}, \tag{5}$$

which indicates that $m_j(t)$ suffers (linear combinations of) exponential decay similar to the equilibrium case. The more interesting case involves baths of *opposite* signs. Then, we can either rely on analytic continuation of $I_n(\alpha t)$ to pure imaginary α or solve equation (4) explicitly. The result involves oscillations with a simple exponential envelope: e^{-t}. As an illustration, if the initial magnetization is homogeneously $\bar{m}$, then

$$m_j(t) = \bar{m}\,\mathrm{e}^{-t}\left[\cos(|\alpha|t) + \frac{\gamma_j}{|\alpha|}\sin(|\alpha|t)\right].$$

Interestingly, the frequency of the oscillations increases as the T's are lowered. Such remarkable properties can perhaps be traced to a mild form of 'frustration', arising from the competition between the two baths. While the effects of the positive T reservoir is to align spins with its neighbours, the other bath struggles to 'anti-align' them. Other notable behaviours occur at the limits. If, say, $T_o \to \infty$ ($\gamma_o \to 0$), then the spins at the odd sites decouple and $m_{2j+1}(t)$ decays purely by e^{-t}. This allows us to integrate (4) for the even sites: $m_{2j}(t) = m_{2j}(0)\,\mathrm{e}^{-t} + \gamma_e t\,\mathrm{e}^{-t}\{m_{2j-1}(0) + m_{2j+1}(0)\}/2$. At first sight, it may seem surprising that the effects of the neighbours linger longer. However, this aspect is due entirely to the details of the dynamics here (random sequential update based on the average spins of the neighbours). At the other extreme, there is qualitatively new behaviour only when *both* T's vanish. As expected, the uniform component of the initial magnetization survives. (As a reminder, note that if the T's $\to 0_-$, only the staggered component remains.)

Equal-time correlations

Next we turn to the time-dependent two-point correlation function: $\langle\sigma_j\sigma_k\rangle_t - \langle\sigma_j\rangle_t\langle\sigma_k\rangle_t$. In most studies of the Ising chain, the second term is typically neglected, since there is generally no spontaneous magnetization. Of course, we have the result for $m_j(t)$ from above and will focus only on $\langle\sigma_j\sigma_k\rangle_t$. The transformation above can still be exploited here, but it is not compatible with the 'boundary condition' $\langle\sigma_k\sigma_k\rangle_t \equiv 1$. Nevertheless, we are able to use the method of images [2] to find the general solution, which is rather involved and will be presented elsewhere [9]. Here, let us illustrate the results by restricting ourselves to a simpler case, namely, one with (period-2) translational invariance (as in [3]). Then, we need to consider only four functions (of one variable: $k - j$), namely, the correlation between spins at two even sites, two odd sites, and one of each. We denote these by $c^{ee}_{2n}(t) \equiv \langle\sigma_{2\ell}\sigma_{2(\ell+n)}\rangle_t$, $c^{oo}_{2n}(t) \equiv \langle\sigma_{2\ell-1}\sigma_{2\ell-1+2n}\rangle_t$, $c^{eo}_{2n-1}(t) \equiv \langle\sigma_{2\ell}\sigma_{2\ell+2n-1}\rangle_t$ and $c^{oe}_{2n-1}(t) \equiv \langle\sigma_{2\ell+1}\sigma_{2\ell+2n}\rangle_t$. Of course, $\langle\sigma_j\sigma_k\rangle_t = \langle\sigma_k\sigma_j\rangle_t$, so that the first pair are even in n, and the last two are related by $c^{eo}_{-2n+1}(t) = c^{oe}_{2n-1}(t)$. Thus, there is no need to study $n < 0$ cases. Finally, we have the boundary condition (BC): $c^{ee}_0 = c^{oo}_0 = 1$, the main source of complication here in comparison with the analysis for $m_j(t)$.

From the master equation (1), we find that these cs satisfy (for $n > 0$)

$$\frac{\mathrm{d}}{\mathrm{d}t}c^{ee}_{2n} = -2c^{ee}_{2n} + \frac{\gamma_e}{2}\left[c^{oe}_{2n-1} + c^{oe}_{2n+1} + c^{eo}_{2n-1} + c^{eo}_{2n+1}\right], \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}c^{oo}_{2n} = -2c^{oo}_{2n} + \frac{\gamma_o}{2}\left[c^{eo}_{2n-1} + c^{eo}_{2n+1} + c^{oe}_{2n-1} + c^{oe}_{2n+1}\right], \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}c^{eo}_{2n-1} = -2c^{eo}_{2n-1} + \frac{\gamma_e}{2}\left[c^{oo}_{2n} + c^{oo}_{2n-2}\right] + \frac{\gamma_o}{2}\left[c^{ee}_{2n} + c^{ee}_{2n-2}\right], \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}c^{oe}_{2n-1} = -2c^{oe}_{2n-1} + \frac{\gamma_o}{2}\left[c^{ee}_{2n} + c^{ee}_{2n-2}\right] + \frac{\gamma_e}{2}\left[c^{oo}_{2n} + c^{oo}_{2n-2}\right]. \tag{9}$$

These simplify considerably, since the combinations $\gamma_e c^{oo}_{2n} - \gamma_o c^{ee}_{2n}$ and $c^{eo}_{2n-1} - c^{oe}_{2n-1}$ decouple and just decay with e^{-2t} from their initial values. Meanwhile, the other combinations, $\gamma_e c^{oo}_{2n} + \gamma_o c^{ee}_{2n}$ and $c^{eo}_{2n-1} + c^{oe}_{2n-1}$ are coupled, but the quantities ($n \geqslant 0$)

$$a_{2n}(t) \equiv \frac{1}{2}\left[\gamma_e c^{oo}_{2n}(t) + \gamma_o c^{ee}_{2n}(t)\right]; \qquad a_{2n-1}(t) \equiv \frac{\alpha}{2}\left[c^{eo}_{2n-1}(t) + c^{oe}_{2n-1}(t)\right], \tag{10}$$

allow us to reduce equations (6)–(9) to a single equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}a_j = -2a_j + \alpha[a_{j-1} + a_{j+1}], \qquad j > 0, \tag{11}$$

with the BC

$$a_0(t) = \bar{\gamma}; \qquad \bar{\gamma} \equiv (\gamma_e + \gamma_o)/2. \tag{12}$$

Now, equations (11), (12) are precisely those encountered by Glauber [2], the only differences being α, $\bar{\gamma}$ instead of γ, 1. An immediate consequence is the steady-state result, which takes the form $a_k(t \to \infty) = \bar{\gamma}\omega_0^k$; $\omega_0 \equiv \tanh[J/k_b T_{\mathrm{eff}}]$, in agreement with those in [3, 4] (λ in [3] $= \omega_0^2$ here). As for the complete solution with arbitrary initial correlations $\langle\sigma_j\sigma_k\rangle_0$, we could simply rewrite Glauber's solution here. Instead, let us illustrate how to derive a more compact form for the time dependence, in a simple example: $\langle\sigma_j\sigma_{k\neq j}\rangle_0 \equiv 0$ (i.e., uncorrelated initial spins if $m_j(0) = 0$ also). Then, $\gamma_e c^{oo}_{2n} - \gamma_o c^{ee}_{2n}$ and $c^{eo}_{2n-1} - c^{oe}_{2n-1}$ ($n > 0$) simply remain zero for all time, so that $c^{ee,oo}_{2n} = a_{2n}/\gamma_{o,e}$ and $c^{eo,oe}_{2n-1} = a_{2n-1}/\alpha$. To see how a_k builds up to the steady-state value, we exploit the Laplace transforms: $\hat{a}_k(s) \equiv \int \mathrm{e}^{-st}a_k(t)$. Condition (12) leads to $\hat{a}_0(s) = \bar{\gamma}/s$, while equations (11) can be solved by an ansatz similar to the one in [3, 4]: $\hat{a}_k(s) = A(s)\omega(s)^k, k > 0$. Inserting these into (11), we find ($A \neq 0$) $\omega^{-1} + \omega = (2 + s)/\alpha$. Meanwhile, $\hat{a}_0(s)$ leads to $A(s) = \bar{\gamma}/s$. As expected, we need only $\omega_0 \equiv \omega(s = 0)$ for the steady state, since the singularities of ω lie at $s \leqslant -2(1 - \alpha)$ and the pole in A controls the $t \to \infty$ limit. For finite t, using properties of Laplace transforms [8], we arrive at a simple result:

$$\langle \sigma_j \sigma_{k\neq j} \rangle_t = \frac{\bar{\gamma}}{\alpha^2} \sqrt{\gamma_j \gamma_k} |j-k| \int_0^{2t} \frac{\mathrm{d}\tau}{\tau} \, \mathrm{e}^{-\tau} I_{|j-k|}(\alpha\tau). \tag{13}$$

With this expression, we can study long-time behaviours of these correlations. Like the case for $m_j(t)$, the leading decay (towards their steady-state values) is monotonic: $t^{-3/2}\,\mathrm{e}^{-2(1-\alpha)t}$. As in the case for $m(t)$, if $T_e T_o < 0, \alpha$ turns pure imaginary, and we find oscillatory behaviour, damped by $t^{-3/2}\,\mathrm{e}^{-2t}$ [9]. Other unusual properties emerge at certain limits. If, for example, $T_o \to \infty$ (T_e finite), then $\alpha, \gamma_o \to 0$. Starting with our initial condition, only the nearest and next-nearest neighbour correlations will build up to non-vanishing values [4]: $c_1(t) = (\gamma_e/4)(1-\mathrm{e}^{-2t})$ and $c_2^{ee}(t) = \left(\gamma_e^2/8\right)[1-\mathrm{e}^{-2t}(1+2t)](c_2^{oo}(t) = 0$, of course). A curious limit is $T_e = -T_o$ in which $\bar{\gamma} = 0$, so that $\langle \sigma_j \sigma_{k\neq j} \rangle_\infty \equiv 0$. Thus, an initially uncorrelated state *never* succeeds in building correlations. We caution that this is a somewhat singular example, as initital correlations are expected to survive with e^{-2t} tails. Finally, we may consider the most extreme case: $T_{e,o} \to 0_{+,-}$. Assuming their magnitudes are unequal in the limiting process, then all correlations are suppressed by $\bar{\gamma} = O(\exp\{-2J/k_b T_>\})$, where $T_>$ is the larger of $T_e, |T_o|$.

General two-point correlation functions

Next, let us ask how a spin on site k at time t is correlated with the spin on site j at a later time $t+t'$. Following Glauber [2] again, we define

$$c_{j,k}(t';t) \equiv \sum_{\{\sigma'\},\{\sigma\}} \sigma'_j \sigma_k \mathcal{P}(\{\sigma'\}, t+t' | \{\sigma\}, t) P(\{\sigma\}, t) \tag{14}$$

where $\mathcal{P}(\{\sigma'\}, t+t'|\{\sigma\}, t)$, is the probability of finding the system with configuration $\{\sigma'\}$ at time $t+t'$ *conditioned* on the configuration being $\{\sigma\}$ at t. Being the propagator for the entire system, $\mathcal{P}$ can be represented as a sum of terms, each of which involves the evolution of N-spin functions: $\left\langle \sigma_{j_1}\sigma_{j_2}\cdots\sigma_{j_N}\right\rangle_t$. For our purposes here, we need only the first two terms: $2^L QTR\mathcal{P}(\{\sigma'\}, t+t'|\{\sigma\}, t) = 1 + \sum_{k,\ell} \sigma'_k \sigma_\ell M_{k\ell}(t') + \cdots$, and arrive at

$$c_{j,k}(t';t) = \sum_\ell M_{j\ell}(t') \langle \sigma_\ell \sigma_k \rangle_t. \tag{15}$$

The interpretation of this formula is clear: all correlations present at time t will be 'propagated' by M over the time delay t' and summed accordingly.

As an illustration, we apply (15) to compute the autocorrelation function $A_{2k,\ell}(t) \equiv c_{2k,\ell}(t,0) - m_{2k}(t) m_\ell(0)$ for a homogeneous system with initial magnetization $\bar{m}$ (and $\alpha \neq 0$). The result is $A_{2k,\ell}(t) = \frac{1-\bar{m}^2}{2\sqrt{\gamma_o}}[\sqrt{\gamma_e} + \sqrt{\gamma_o} + (-1)^\ell(\sqrt{\gamma_o} - \sqrt{\gamma_e})]\,\mathrm{e}^{-t} I_{2k-\ell}(\alpha t)$. This simple example shows that the amplitude of the autocorrelation function alternates with the parity of site ℓ and depends on the temperatures.

Higher correlation functions

Finally let us turn to equal-time correlations of N spins: $\left\langle \sigma_{j_1} \ldots \sigma_{j_N} \right\rangle_t$. For the specific case of an initially uncorrelated system with zero magnetization, all functions with odd N vanish, of course. For $N = 2n$, we show that they can all be expressed in terms of two-spin functions (13).

This programme is achieved by taking advantage of recent formal results obtained by Aliev [10]. He considered a completely general version of the kinetic Ising chain, with $1-\tilde{\sigma}_j(c_j\sigma_{j-1}+\mathrm{d}_j\sigma_{j+1})/2$ instead of $[1-\gamma_j\tilde{\sigma}_j(\sigma_{j-1}+\sigma_{j+1})/2]$ in the spin-flip rates (2). Such a model would correspond to a system with not only arbitrary nearest-neighbour couplings ($J_{k,k+1}$), but also a separate bath (at T_k) for each spin! In the absence of initial magnetization and correlations, Aliev was able to show that the generating function for all correlations, $\Psi(\{\eta\};t)\equiv\left\langle\prod_j(1+\eta_j\sigma_j)\right\rangle_t$, where the η's are Grassmannian variables [11], is given by $\Psi=\exp\left(\sum_{j<k}\eta_j\eta_k\langle\sigma_j\sigma_k\rangle_t\right)$. Using the notation of *Pfaffians* [11, 12], we can expand Ψ and arrive at an expression for $2n$-point function ($j_1<j_2<\cdots<j_{2n}$):

$$\left\langle\sigma_{j_1}\sigma_{j_2}\cdots\sigma_{j_{2n-1}}\sigma_{j_{2n}}\right\rangle_t=\sum_\pi\frac{(-1)^{\mathrm{Sg}\pi}}{n!}\left\langle\sigma_{j_{\pi(1)}}\sigma_{j_{\pi(2)}}\right\rangle_t\ldots\left\langle\sigma_{j_{\pi(2n-1)}}\sigma_{j_{\pi(2n)}}\right\rangle_t,\qquad(16)$$

where the summation runs over all the permutations π of the indices $\{j_1,j_2,\ldots,j_{2n-1},j_{2n}\}$, with the constraint that $j_{\pi(2\ell-1)}<j_{\pi(2\ell)}$ for each ℓ. Here, $\mathrm{Sg}\pi$ is the signature of the permutation π. For example, the general time-dependent four-spin correlation is given by ($i<j<k<\ell$): $\langle\sigma_i\sigma_j\sigma_k\sigma_\ell\rangle_t=\langle\sigma_i\sigma_j\rangle_t\langle\sigma_k\sigma_\ell\rangle_t-\langle\sigma_i\sigma_k\rangle_t\langle\sigma_j\sigma_\ell\rangle_t+\langle\sigma_i\sigma_\ell\rangle_t\langle\sigma_j\sigma_k\rangle_t$. In the steady state, the last two terms cancel. More generally, such cancellations can be shown to persist for arbitrary n, so that we recover the result of [4]: $\left\langle\sigma_{j_1}\ldots\sigma_{j_{2n}}\right\rangle_\infty=\left\langle\sigma_{j_1}\sigma_{j_2}\right\rangle_\infty\ldots\left\langle\sigma_{j_{2n-1}}\sigma_{j_{2n}}\right\rangle_\infty$.

Before closing, we remind the readers that the results in this section hold only for an initially uncorrelated state with zero magnetization. Otherwise, the N-point functions will clearly be more complex than (16). Finally, due to equation (3), we see that the full distribution $P(\{\sigma\},t)$ for this specific nonequilibrium many-body problem can be constructed from (13) and (16).

Concluding remarks

In this letter we solved a stochastic Ising chain in which alternate spins are coupled to two thermal baths at different temperatures via Glauber spin-flip dynamics. We found analytic expressions for all correlation functions. If both temperatures are positive, both the steady state and the decays into it display properties similar to those in the ordinary Glauber–Ising model. If the temperatures are of opposite signs, qualitatively novel behaviours, such as oscillatory damping, emerge. Similar to the spatial oscillations in stationary two-spin correlations, we believe their origins can be thought of as a kind of ‘frustration’, where the two baths attempt to align/anti-align a spin with its neighbours.

Finally, we note that our findings can be applied to studies of the dynamics of *domain walls* in this system [5]. As usual, the kinetic Ising model can be mapped onto a reaction–diffusion system (RDS), in which a ‘particle’ (A) corresponds to a broken bond on the Ising lattice [1]. The resulting RDS, in addition to symmetric diffusion (with rate 1), would be pair-annihilation ($AA\to\emptyset\emptyset$) with rate $1+\gamma_j$ and pair-creation ($\emptyset\emptyset\to AA$) with rate $1-\gamma_j$. Since $|\gamma_j|\leqslant 1$, we are satisfied that these rates are positive and, so, physical. Our two-temperature model is then mapped onto an RDS with two different creation/annihilation rates on alternating sites. The implications of mapping our results onto the RDS case are interesting, e.g., *oscillatory* damping of the density of domain-walls when $\gamma_e\gamma_o<0$. Further details will be published elsewhere [9].

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